MATH 448 — Homework 1

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1. A laboratory fly population has fecundity $f = 0.12$ and death rate $d = 0.05$ per day, and starts at $P_0 = 400$ flies.

(a) Write ΔP in terms of P_t.

(b) Write P_{t+1} in terms of P_t and give λ.

(c) Give an explicit formula for P_t and compute P_5.

2. (Allman & Rhodes, Problem 1.1.12.) Suppose the size of a certain population is affected only by birth, death, immigration, and emigration — each of which occurs in a yearly amount proportional to the size of the population. That is, if the population is P, then within a time period of one year the number of births is bP, the number of deaths is dP, the number of immigrants is iP, and the number of emigrants is eP, for some b, d, i, and e. Show that the population can still be modeled by $\Delta P = rP$, and give a formula for r.

3. A salmon population reproduces once per generation: each female lays 3000 eggs and then dies, and a fraction s of the eggs survive to become spawning females. Measure t in generations.

(a) What is d?

(b) Write the model and give λ in terms of s.

(c) Find the value of s for which the population is stable.

(d) If $s = 0.001$, does the population grow or decline, and by what factor per generation?

4. (Allman & Rhodes, Problems 1.1.15 and 1.1.16(a)–(d).) These ask you to rewrite the model $N_{t+1} = 2N_t$ under a change of population units (1.1.15), and under a change of the time step (1.1.16).

5. (Data from Allman & Rhodes, Table 1.2.) An insect population was measured in a laboratory experiment over eleven consecutive days:

t	0	1	2	3	4	5	6	7	8	9	10
P_t	.97	1.52	2.31	3.36	4.63	5.94	7.04	7.76	8.13	8.3	8.36

(a) Compute the ratio P_{t+1}/P_t for each t.

(b) Would a model $P_{t+1} = \lambda P_t$ with a single constant λ describe this population over all eleven days? Justify your answer from the ratios, not from a plot alone.

(c) Over which days would it do well, and what is λ there?

(d) Describe in one sentence what is happening to the per-capita growth rate as the population grows.

6. Let $r = 0.4$, $K = 500$, and $P_0 = 60$ in the discrete logistic model.

(a) Write the model explicitly.

(b) Compute P_1, P_2, P_3.

(c) Compute the per-capita growth rate $\Delta P/P$ at P_0 and at P_2, and confirm that it decreased.

7. In a population obeying the discrete logistic model, the per-capita growth rate is measured to be 0.36 when $P = 100$ and 0.12 when $P = 400$. Find r and K. What is the per-capita growth rate at $P = 550$, and what does its sign mean?

8. Let $r = 1$ and $K = 10$, so that $P_{t+1} = P_t(2 - P_t/10)$.

(a) Find where the graph of F meets the horizontal axis and where its vertex is, and sketch F together with the diagonal over $0 \le P \le 20$.

(b) Cobweb four steps starting from $P_0 = 4$.

(c) Check your picture against the computed values 6.4, 8.704, 9.832.

9. Let $r = 2.2$ and $K = 10$, with $P_0 = 5$. Compute P_1 through P_5 to three decimals, and describe the behavior in words. (Explaining this behavior is not expected; it is next week's material.)

10. Let $r = 1.5$ and $K = 10$.

(a) Find every population size P_t for which the model predicts $P_{t+1} < 0$.

(b) Compute P_1 when $P_0 = 20$.

(c) In two or three sentences: is "a negative population means extinction" a satisfactory reading, and what property should a better model have? (Section 1.4 gives one such model; we have not covered it. Read §1.4 up to the Ricker formula before answering this part.)