MATH 448 — Homework 2

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Exercise 3.1. Let $r = 1.2$ and $K = 8$ in the discrete logistic model. Find the equilibria and classify each as stable or unstable using the calculus test.

Exercise 3.2. Using the bifurcation diagram from Lecture 3: (a) estimate the value of r at which the population first settles into a 2-cycle rather than a single equilibrium; (b) at $r = 2.7$, roughly how many distinct long-run values does the population appear to take, and what does that suggest about the behavior there?

Exercise 3.3. Let $r = 2.75$, $K = 1$ (the chaotic case from Lecture 3), so $P_{t+1} = P_t + 2.75P_t(1 - P_t)$. By calculator, compute P_0 through P_{13}, rounding to four decimal places at each step, starting from (a) $P_0 = 0.5$, and separately from (b) $P_0 = 0.5001$. (c) At what step do the two sequences first differ by more than 0.01? (d) Is that divergence a sign of an arithmetic mistake somewhere, or is it an expected feature of the model itself? Explain in one or two sentences.

Exercise 3.4. For the discrete logistic model with $K = 10$, state whether you expect *monotone approach*, *damped oscillation*, or a *stable cycle* at $P^* = K$ for each growth rate below, and justify your answer using the sign and magnitude of the stretching factor $1 - r$: (a) $r = 0.4$; (b) $r = 1.6$; (c) $r = 2.2$.

Exercise 3.5. True or false, with a one-sentence justification: chaotic behavior in the logistic model means the population's future values are essentially random.