MATH 448 — Homework 3

Exercise 4.1. Let $r = 0.5$ and $K = 20$ in the Ricker model. Find the equilibria and determine whether $P^* = K$ is stable.

Exercise 4.2. Populations subject to a strong Allee effect decline at low density instead of growing (a minimum viable density is needed before reproduction outpaces mortality). One such model is

$$\frac{dP}{dt} = rP\Big(\frac{P}{A} - 1\Big)\Big(1 - \frac{P}{K}\Big), \qquad 0 < A < K,$$

where A is the Allee threshold. Let $r = 1$, $A = 2$, $K = 8$.

(a) Find all equilibria.

(b) Using the phase line, determine the sign of dP/dt on each of the intervals $(0, A)$, (A, K), and (K, ∞).

(c) Classify each equilibrium as stable or unstable. What does the result at $P = 0$ mean biologically for a population that starts below the Allee threshold?

Exercise 4.3. True or false, with a one- or two-sentence justification: a system of two coupled continuous-time differential equations (such as a predator–prey model) can exhibit chaotic behavior.

Exercise 5.3 (*computer exploration*). Euler's method turns the logistic ODE into $P_{t+1} = P_t\big(1 + hr(1 - P_t/K)\big)$ (Problem 5.1), which is exactly the discrete logistic model with growth parameter hr in place of r.

(a) Using the discrete logistic model's own stability threshold (Lectures 2–3), find the critical step size h^* above which the Euler simulation of $P^* = K$ becomes unstable, for $r = 2.5$.

(b) The MATLAB code below simulates Euler's method for the logistic ODE:

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r = 2.5; K = 10; P0 = 1; N = 40; h = 0.1; % change h here
P = zeros(1, N+1);
P(1) = P0;
for t = 1:N
    P(t+1) = P(t) + h*r*P(t)*(1 - P(t)/K);
end
plot(0:N, P, 'o-')
xlabel('t'); ylabel('P(t)')
```

Run it for $h = 0.1$, 0.5, 1, and 1.5. For each, describe what happens to P_t: does it approach K smoothly, oscillate, or diverge?

(c) Do the simulations in (b) match the threshold you found in (a)? Which value of h reproduces the oscillating case shown in the Lecture 5 figure?

Exercise 5.4. In one or two sentences, explain the difference between *fitness* and *payoff* as used in the replicator equation.