MATH 448 — Homework 4

Exercise 5.1. Using the derivative test instead of the phase line, confirm that $x^* = 2/3$ is stable when $V = 4$, $C = 6$.

Exercise 5.2. In the *Stag Hunt* game, two strategies are available: *Stag* (cooperate on a large, risky prey) and *Hare* (a smaller prey caught alone regardless of the other player). The payoff matrix is

	Stag	Hare
Stag	4	0
Hare	3	3

Let x be the frequency of Stag.

(a) Find $f_{\text{Stag}}(x) - f_{\text{Hare}}(x)$ and write down the replicator equation.

(b) Find all equilibria and classify the stability of each (phase line or derivative test, your choice).

(c) Compare your result to Hawk–Dove from Lecture 5: is the long-run outcome here the same kind of result (a single attracting mix) or qualitatively different? Explain in one or two sentences.

Exercise 6.1. Let $r = 0.8$ and $K = 12$. Find the maximum sustainable constant harvest H^*, and find both equilibria when $H = 2$.

Exercise 6.2. Let $r = 0.8$ and $K = 12$ as in Exercise 6.1. Find the maximum sustainable harvest fraction h^*, and find the equilibrium population when $h = 0.5$.

Exercise 7.1 (*computer exploration*). The MATLAB code below builds the loggerhead sea turtle projection matrix from Lecture 7 and finds its eigenvalues:

```
L = [0 0 0 0 127 4 80 ;
     0.6747 0.7370 0 0 0 0 0 ;
     0 0.0486 0.6610 0 0 0 0 ;
     0 0 0.0147 0.6907 0 0 0 ;
     0 0 0 0.0518 0 0 0 ;
     0 0 0 0 0.8091 0 0 ;
     0 0 0 0 0 0.8091 0.8089];
eigenvalues = eig(L)
[V, D] = eig(L);
```

Run it and find the dominant eigenvalue. Is the population growing or declining? Now increase the survival entry for large juveniles (row 3, column 3, currently 0.6610) and re-run. Roughly how large an increase is needed before the dominant eigenvalue exceeds 1?