MATH 448 — Homework 5 — Solutions

Exercise 8.1. The matrix is $A = \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix}$, with

$$T = -2 + (-4) = -6, \qquad D = (-2)(-4) - (3)(1) = 8 - 3 = 5.$$

Since $D > 0$ and $T < 0$, the equilibrium is stable; since $T^2 - 4D = 36 - 20 = 16 > 0$, the eigenvalues are real, so this is a **stable node**. (Directly: $\lambda^2 + 6\lambda + 5 = 0$ factors as $(\lambda + 1)(\lambda + 5) = 0$, giving $\lambda = -1, -5$, both real and negative.)

Exercise 8.2. For $A = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$:

$$T = 2, \qquad D = 1 - 0.25 = 0.75, \qquad T^2 - 4D = 4 - 3 = 1 > 0.$$

Real eigenvalues ($\lambda = 1.5,\ 0.5$, both positive), $T > 0$: an **unstable node**.
For $A = \begin{pmatrix} 0.2 & 1 \\ -1 & 0.2 \end{pmatrix}$:

$$T = 0.4, \qquad D = (0.2)(0.2) - (1)(-1) = 0.04 + 1 = 1.04, \qquad T^2 - 4D = 0.16 - 4.16 = -4 < 0.$$

Complex eigenvalues ($\lambda = 0.2 \pm i$), real part $T/2 = 0.2 > 0$: an **unstable spiral**. Running the code should show trajectories spiraling outward in both cases, more slowly for the second (smaller $|T|$) than the first.