MATH 448 — Homework 6 — Solutions

Exercise 9.1. The coexistence equilibrium is

$$(x^*, y^*) = \left(\frac{\gamma}{\delta},\ \frac{\alpha}{\beta}\right) = \left(\frac{1}{0.5},\ \frac{2}{1}\right) = (2, 2).$$

The Jacobian there is

$$J = \begin{pmatrix} 0 & -\beta\gamma/\delta \\ \alpha\delta/\beta & 0 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}, \qquad T = 0, \qquad D = \alpha\gamma = 2.$$

Since $T = 0$ and $D > 0$, this is a **center**, as it must be for every choice of positive parameters (Problem 9.2).

Exercise 9.2. Each initial population lies on a different level curve of the conserved quantity H from Problem 9.3, so each traces its own closed loop, nested around $(2, 2)$ without crossing the others. Running the code with $\alpha = 2, \beta = 1, \gamma = 1, \delta = 0.5$ and, for instance, $(x_0, y_0) = (3, 1)$, gives a period of about 4.7 time units, read off as the spacing between successive peaks of $x(t)$ in the time-series panel. This is close to, but not exactly, the small-oscillation estimate $2\pi/\sqrt{\alpha\gamma} \approx 4.44$ from the linearization — expected, since the linearization is only exact in the limit of small oscillations around the equilibrium, and larger orbits are further from that limit.