MATH 448 — Homework 8

A reaction network *computes* a function f if, for every starting input (placed in the tube with nothing else) and every order in which the reactions happen, the process stops, and when it stops the tube contains exactly $f(\cdot)$ molecules of Y. For two inputs, x_1 molecules of X_1 and x_2 molecules of X_2 are placed in the tube. Other species may remain in the tube at the end.

Exercise 10.1. The function $f(x_1, x_2) = x_1 - x_2$ makes sense as a count of molecules when $x_1 \geq x_2$. A student proposes the network with two reactions $X_1 + X_2 \to \varnothing$ and $X_1 \to Y$ to compute it. Find starting counts with $x_1 \geq x_2$ and an order of reactions for which the tube ends with the wrong number of Y's.

Exercise 10.2. Design a network that computes $f(x_1, x_2) = x_1 - x_2$ whenever $x_1 \geq x_2$, and explain why it works. (Worksheet Problem 6 may help.)

Exercise 10.3. Show that the network $X_1 + X_2 \to Y$ computes $f(x_1, x_2) = \min(x_1, x_2)$ for every x_1 and x_2. *Hint:* When the reaction happens, what happens to (number of X_1) − (number of X_2)? What happens to (number of X_1) + (number of Y)?

Exercise 10.4. Design a network that computes the function with $f(x) = 1$ when x is odd and $f(x) = 0$ when x is even, and explain why it works.

Exercise 10.5. Design a network that computes the function with $f(x) = 1$ when $x \geq 1$ and $f(x) = 0$ when $x = 0$, and explain why it works.

Exercise 10.6. Explain why no network computes $f(x) = x + 1$. *Hint:* Consider $x = 0$.